FSF July Math Competition Elementary School Exam

July 2024

1. Evaluate: $2 - 2(2 + 2)/2$

   (a) 2
   (b) -2
   (c) 4
   (d) 0

   Solution: $2 - 2(2 + 2)/2 = 2 - 2(4)/2 = 2 - 4 = \boxed{\text{-2}}$.

2. What is the sum of the first 20 odd numbers?

   (a) 400
   (b) 410
   (c) 424
   (d) 438

   Solution: The sum of the first n odd numbers is equal to n^2. Therefore, the sum of the first 20 odd numbers is 20^2 or $\boxed{400}$.

3. What is the next number in the sequence with the first 5 terms $0, 1, 2, 5, 26$?

   (a) 59
   (b) 82
   (c) 127
   (d) 677

   Solution: The sequence can be represented by the equation $a_n = a_{n-1}^2 + 1$. Therefore, the next term in the sequence would be $26^2 + 1$ or $\boxed{677}$.

4. On a specific math test, the probability of you answering the first problem right is 1, the probability of answering the second problem right is 1/2, third problem is 1/3, and the nth problem is $1/n$. What is the probability you get all of the first 5 problems right?

   (a) 3/400
   (b) 1/120
   (c) 1/60
   (d) 5/12

   Solution: The probability of getting all of the first five problems right would be the product of getting each individual question right. Therefore, it would be $1 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} = \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = \boxed{1/120}$.

5. A rectangular prism has a volume of 880, what is the smallest possible sum of its edges?

   (a) 96
   (b) 116
   (c) 136
   (d) 156

   Solution: The smallest possible sum of its edges would mean that they are as close to each other as possible. In this case, that would be 8, 10, and 11. Since this is a rectangular prism, each edge length appears 4 times (there are 12 edges total) so the the sum of its edges would be $4(8 + 10 + 11) = \boxed{116}$.

6. In a farm, there are chickens and pigs. If there are a total of 24 heads and 76 legs, how many chickens are there?

   (a) 8
   (b) 10
   (c) 12
   (d) 16

   Solution:

   Let there be x chickens and y dogs, we have $x + y = 24, 2x + 4y = 76$. We want to solve for x. Multiply the first equation by 4 gives $4x + 4y = 96$. Subtract the 2nd equation from that gives $2x = 20$ so $x = \boxed{10}$.

7. How many ways are there for 5 people Alice, Bob, Cindy, David, and Emma to stand in a line such that David must stand in front of Emma?

   (a) 20

   (b) 24

   (c) 30

   (d) 60

   Solution: Note that the number of ways to arrange the 5 people in a line where David is in front of Emma is the same as the number of ways to arrange such that Emma is in front of David. Since the total of these 2 add up to the total number of ways to arrange the people in a line. The answer is just $\frac{5!}{2}$ or $\boxed{60}$.

8. I will roll a die until I get a prime number, what is the probability that after 6 rolls I will still need to roll a 7th time?

   (a) 1/32

   (b) 1/64

   (c) 1/128

   (d) 1/256

   Solution: Needing a 7th roll means that you rolled 6 composite numbers in the first 6 rolls. Ther are 3 composites on a die so the answer is $(\frac{1}{2})^6$ or $\boxed{1/64}$.

9. How many ways are there to arrange 2 red circles, 4 green circles, and 5 blue circles in a larger circle if the same colored circles are indistinguishable?

   (a) 2520

   (b) 7560

   (c) 40320

   (d) 362880

   Solution: The number of ways to arrange n circles in a circle is $(n-1)!$. Since same colored circles are indistinguishable, answer is $10!/(2! \cdot 4! \cdot 5!)$ or $\boxed{630}$. (The answer choices are wrong)

10. A positive integer leaves a remainder of 2 when divided by $3, 4$, and 5. What is the 2nd smallest possible value for the number?

    (a) 2

    (b) 32

    (c) 62

    (d) 82

    Solution: The smallest possible positive integer is obviously 2, we want the 2nd smallest. Since it leaves a remainder of 2, when divided by $3, 4, 5$. It will leave a remainder of 0 when the number minus 2 is divided by $3, 4, 5$. The 2nd smallest will be 2 more than the least common multiple of $3, 4, 5$ which is $\boxed{62}$.

11. What is the probability that a randomly selected positive integer leaves a remainder of 4 when divided by 11?

    (a) 1/2

    (b) 1/4

    (c) 4/11

    (d) 1/11

    Solution: Note that for every 11 positive integers, there is 1 that leaves remainder of 4 when divided by 11. So, the answer is just $\boxed{1/11}$.

12. In a box, there are 5 red balls, 20 green balls, 45 blue balls, and 103 yellow balls. Without looking in the box and not knowing the ball you draw, how many balls do you need to draw to be sure that you have drawn a red ball?

    (a) 4

    (b) 70

    (c) 167

    (d) 168

    Solution: The worst case is when you draw every other colored ball and still haven't drawn a red ball. That is $20 + 45 + 103 = \boxed{168}$. Now, you are sure you will draw a red ball.

13. A palindrome is a number that reads the same left to right. How many 5 digit palindromes are there such that its digits are all odd or all even?

    (a) 125
    (b) 225
    (c) 250
    (d) 300

    Solution: If the digits are all odd, there are 5 options for the ten-thousands and ones digit (they're the same), 1, 3, 5, 7, and 9. The same applies for the thousands and tens digit, and also the hundreds digit. There are three digits total to pick, and there are 5 options for each digit, therefore there are $5 \cdot 5 \cdot 5 = 125$ palindromes with all odd digits. The same applies for even digits, where the possible digits can be 0, 2, 4, 6, and 8. However, the ten-thousands digit cannot be 0, therefore there are 4 options for the ten-thousands and ones digit and 5 options for the others. Therefore, there are $4 \cdot 5 \cdot 5 = 100$ palindromes with all even digits. Adding these up, there are a total of $\boxed{225}$ palindromes with all odd or all even digits.

14. How many positive integers less than 1000 have exactly 5 factors?

    (a) 1
    (b) 3
    (c) 10
    (d) 60

    Solution: The only numbers with exactly 5 factors are primes to the 4th power. So, the only numbers less than 1000 that are prime to 4th power are $2^4, 3^4, 5^4$ so total is $\boxed{3}$.

15. I pick 2 points A and B on the circumference of a circle with diameter 12. What is the probability that $AB < 6$?

    (a) 1/6
    (b) 1/4
    (c) 1/3
    (d) 1/2

    Solution: Since the diameter of the circle is 12, the radius is 6. In order for $AB < 6$, the measure of AB must be less than or equal to $60°$. For anywhere that we put A on the circle, you can place B up to $60°$ to the right and up to $60°$ to the left of A. This adds up to $120°$, and since there are $360°$ total in a circle, the probability that $AB < 60°$ or $AB < 6$ is $\frac{120°}{360°}$, or $\boxed{1/3}$.

END OF TEST