

# FSF Math Competition Elementary School Exam

February 2025

1. Evaluate  $2(3(4 + 5))$ .

- (a) 30
- (b) 36
- (c) 48
- (d) 54 ← CORRECT

Solution:  $2(3(4 + 5)) = 2 * 3 * 9 = \boxed{54}$

2. Evaluate  $\frac{2^5}{4^2}$ .

- (a) 1
- (b) 2 ← CORRECT
- (c) 4
- (d) 8

Solution:  $\frac{2^5}{4^2} = 32/16 = \boxed{2}$

3. How many seconds are in 1 hour and 52 minutes?

- (a) 5040
- (b) 6720 ← CORRECT
- (c) 8032
- (d) 10240

Solution: The number of seconds is  $60 * 60 + 52 * 60 = 112 * 60 = \boxed{6720}$

4. Suppose David got a 60 out of 100 on his first chemistry test. If he scores 100 out of 100 on every test after that, how many tests does he need to take to bring his average test score to a 90 out of 100?

- (a) 3 ← CORRECT
- (b) 5
- (c) 7
- (d) 9

Solution: Let  $x$  be the number of tests he must take we have  $\frac{60+100x}{x+1} = 90 \rightarrow 90x+90 = 100x+60 \rightarrow x = \boxed{3}$ .

5. What is the next number in the sequence: 3, 4, 7, 12, 19?

- (a) 23
- (b) 26
- (c) 28 ← CORRECT
- (d) 31

Solution: Notice that the different between 2 consecutive terms have the sequence 1, 3, 5, 7, ... So the next one is +9,  $19 + 9 = \boxed{28}$ .

6. By definition,  $(x\delta y) = x - y$ . What is  $9\delta(6\delta 3)$ ?

- (a) 1
- (b) 3
- (c) 6 ← CORRECT
- (d) 9

Solution: we have  $9\delta(6\delta 3) = 9\delta(6 - 3) = 9\delta 3 = 9 - 3 = \boxed{6}$

7. Adam is rowing a boat. When rowing downstream, he is moving at a speed of 6 mph. When rowing upstream, he is moving at a speed of 4 mph. Supposing that he rows at the same rate both times, what would his speed be if he was rowing at that same rate in still water?

- (a) 4.25
- (b) 4.5
- (c) 4.75
- (d) 5 ← CORRECT

Solution: Let the speed of boat be  $x$  and speed of current be  $y$ .  $x + y = 6, x - y = 4 \rightarrow x = 5, y = 1$ . So the answer is  $\boxed{5}$ .

8. Alice, Bob, Claire, and David are sitting in a row, with seats numbered 1, 2, 3, and 4. It is known that David is sitting in an even number seat and that Bob and Clair are sitting in prime number seats. What seat is Alice sitting in?

- (a) 1 ← CORRECT
- (b) 2
- (c) 3
- (d) 4

Solution: Since 2 and 3 are the only primes that's where Bob and Clair are sitting. Then since David is even number he is 4. That leaves Alice at  $\boxed{1}$

9. How many of the first 10 even numbers can be expressed as a sum of 2 prime numbers?

- (a) 3
- (b) 5
- (c) 9 ← CORRECT
- (d) 10

Solution: We can find a construction for all of them except 2:  $4 = 2 + 2, 6 = 3 + 3, 8 = 5 + 3, 10 = 5 + 5, 12 = 7 + 5, 14 = 7 + 7, 16 = 11 + 5, 18 = 11 + 7, 20 = 13 + 7$ . This is also known as the goldbach's conjecture.

10. How many ways are there to arrange 4 people in a line of 6 chairs?

- (a) 60
- (b) 120
- (c) 240
- (d) 360 ← CORRECT

Solution: The first person has 6 places to sit, 2nd has 5, 3rd has 4, and 4th has 3. So its  $6 * 5 * 4 * 3 = \boxed{360}$ .

11. Let  $\text{LCM}(x, y) = 546$ . If the greatest prime factor of  $x$  is 7, what is the greatest prime factor of  $y$ ?

- (a) 2
- (b) 3
- (c) 7
- (d) 13 ← CORRECT

Solution:  $546 = 2 * 3 * 7 * 13$ . Since  $x$  doesn't contribute a 13 in the prime factorization,  $y$  must contribute it, so the answer is  $\boxed{13}$ .

12. Alice makes an 8 by 8 square grid out of matchsticks, where each matchstick is one unit. How many matchsticks does she use total?

- (a) 32
- (b) 64
- (c) 144 ← CORRECT
- (d) 256

Every square has 4 matchsticks, so  $64 * 4 = 256$ . But that overcounts. Except the matchsticks on the border, all other matchsticks are counted twice. Only the  $4 * 8 = 32$  matchsticks on the border aren't counted twice. Let  $x$  be the total number of matchsticks.  $x - 32$  are counted twice. We have  $2(x - 32) + 32 = 256 \rightarrow x - 32 = 112 \implies x = \boxed{144}$ .

13. Let Grace have two bottles of fruit punch: bottle  $A$  is  $1/5$  orange juice, and bottle  $B$  is  $1/3$  orange juice. If Grace wants to mix a certain amount of punch from these bottles to make a new fruit punch that is  $1/4$  orange juice, what fraction of the new punch is from bottle  $A$ ?
- (a)  $1/5$
  - (b)  $5/8$  ← CORRECT
  - (c)  $3/4$
  - (d)  $4/9$

Solution: Let  $x$  be the fraction of punch from bottle  $A$ . We have

$$\frac{x}{5} + \frac{1-x}{3} = \frac{1}{4} \rightarrow \frac{5-2x}{15} = \frac{1}{4} \rightarrow 20-8x=15 \rightarrow x = \boxed{\frac{5}{8}}.$$

14. There are 5 students taking a math test at the FSF high school. If the school has two classrooms that can each hold at most 5 students, how many ways are there to arrange the students into classrooms? (Assume the classrooms are distinguishable and the students are indistinguishable).
- (a) 2
  - (b) 4
  - (c) 6 ← CORRECT
  - (d) 8

Solution: Let the rooms be  $A, B$  and the number of students in each room  $A$  and  $B$  be denoted as  $(x, y)$  where  $x$  is number in  $A$  and  $y$  is number in  $B$ . The possible combinations are  $(5, 0), (4, 1), (3, 2), (2, 3), (1, 4)$ .  $(0, 5) \rightarrow \boxed{6}$ .

15. An ant is on a pentagon with vertices labeled 1, 2, 3, 4, and 5, in order. The ant has equal probability to move forward 1 step (at point 1, moving back one step goes to point 5). What is the probability the ant reaches point 4 in three steps if it starts at point 1?
- (a)  $1/8$  ← CORRECT
  - (b)  $1/6$
  - (c)  $1/4$
  - (d)  $1/3$

Solution: In 3 steps the only possible way is to go from 1 to 2 to 3 to 4. There are no other possible ways because if you go back 2 and forward 1, you land on 5. Back 1 forward 2 gives 2, and back 3 gives 3. So its  $\boxed{1/8}$ .

16. **Free Response Question.** Terence's basketball team won 60 percent of the first 20 games they played this season. Then, they won 70 percent of the remaining 10 games. What was their total winning rate this season? (Winning rate means the amount they won divided by the total number of games they played. Answers can be either fractions or percents.)

Solution: 60% of 20 is 12, and 70% of 10 is 7. That means that they won a total of  $12 + 7 = 19$  games this season, and they played a total of 30 games. Therefore, the total winning rate is  $\boxed{19/30}$  or  $\boxed{63.33\%}$ .

END OF TEST